Random Graphs
Exercise Sheet 1

Question 1. Give an example of a set of events $\{A_i : i \in I\}$ which are pairwise independent, but not mutually independent.

Give an example of a collection of random variables $\{X_i : i \in I\}$ which are pairwise independent, but not mutually independent.

Give an example of two random variables X and Y such that $\mathbb{E}(XY) \neq \mathbb{E}(X)\mathbb{E}(Y)$.

Question 2. Calculate the following probabilities:

- $\mathbb{P}(G(4, 1/2)$ has 2 edges);
- $\mathbb{P}(G(4, 1/2)$ has 6 edges);
- $\mathbb{P}(G(4, 1/2)$ is connected).

Question 3. Let $\left(G_1, \ldots, G_{\binom{n}{2}}\right)$ be the sequence of random variables given by the random graph process in the lecture. Show that $G_m \sim G(n, m)$ for each m.

Question 4. Show that with high probability $\mathbb{P}(G(n, p)$ has diameter $\leq 2)$, for constant p.

Question 5. Show directly, that is without using any relation between $G_{n,m}$ and $G_{n,p}$, that every monotone graph property has a threshold in $G_{n,m}$.

(Hint: It may be useful to relate the random variable $G_{n,km}$ to the random variable given by the union of k independent copies of $G_{n,m}$).

Question 6 (Harris' inequality)**.** Let X and Y be increasing graph parameters, i.e., $X(G) \leq X(H)$ whenever $G \subseteq H$. Show that

$$\mathbb{E}_{G(n,p)}(XY) \geq \mathbb{E}_{G(n,p)}(X)\mathbb{E}_{G(n,p)}(Y).$$

(Hint: Induct on the number of edges which determine the value of X and Y)